Bank Runs and Systemic Risk

— Week 4 —

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Summary

1. Recall some numbers
2. Attitudes towards risk
3. Introduction to game theory
4. The Diamond-Dybvig bank run model
5. Bibliography

I - Recall some numbers

The balance sheet of commercial banks in the US

Assets	$ Billion	% Total	Liabilities and Net Worth	$ Billion	% Total
Real assets			**Liabilities**		
Equipment and premises	$ 100.7	1.0%	Deposits	$ 6,865.3	65.9%
Other real estate	6.8	0.1	Borrowed funds	1,242.5	11.9
Total real assets	$ 107.5	1.0%	Subordinated debt	161.3	1.5
			Federal funds and repurchase agreements	771.4	7.4
			Other	320.8	3.1
			Total liabilities	$ 9,361.3	89.9%
Financial assets					
Cash	$ 457.5	4.4%			
Investment securities	2,180.0	20.9			
Loans and leases	6,089.3	58.5			
Other financial assets	822.3	7.9			
Total financial assets	$ 9,549.1	91.7%			
Other assets					
Intangible assets	$ 379.2	3.6%			
Other	375.1	3.6			
Total other assets	$ 754.3	7.2%	*Net worth*	$ 1,049.6	10.1%
Total	$ 10,410.9	100.0%		$10,410.9	100.0%

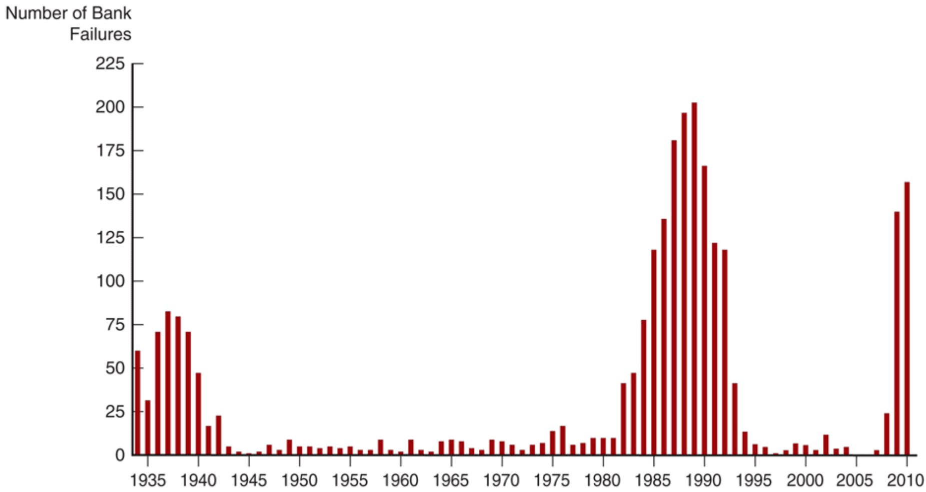
TABLE 1.3

Balance sheet of commercial banks, 2007

Note: Column sums may differ from total because of rounding error.

Source: Federal Deposit Insurance Corporation, **www.fdic.gov**, September 2007.

Systemic risk in the US



Do we really need banks?

1. With so many problems
   1. Low reserve/deposits ratios
   2. Low capital ratios
   3. Maturities mismatch: liabilities (short run) vs assets (long run)
   4. The principal vs agent problem
   5. The Too-Big-Too-Fail problem
2. Do we really need banks?
3. Surprising, the answer is : YES, WE DO. BUT ...
4. That's what Douglas Diamond and Philip Dybvig (1983) demonstrated.

Douglas Diamond and Philip Dybvig (1983). Bank Runs, Deposit Insurance, and Liquidity, *The Journal of Political Economy*, 91(3), 401–419.

Diamond–Dybvig model: basics

1. It explains the emergence of banks as a **risk sharing** agreement between depositors against **unexpected liquidity needs**
2. If people are risk averse, they will be better off with banks
3. The model shows also that bank runs are highly possible, as an act of collective irrationality by rational depositors
4. Can a "bank run" occur only if **bankers are irresponsible**? NO.
5. Shows the usefulness of deposit insurance as an efficient mechanism to prevent bank runs

Douglas Diamond (2007). Banks and Liquidity Creation: A Simple Exposition of the Diamond-Dybvig Model, *Federal Reserve Bank of Richmond Economic Quarterly*, 93(2), 189–200. `Read this paper`

"Fundamentals" versus panic: an example

1. Why did Bear Stearns, Lehman Brothers in the US, or Northern Rock in the UK, suddenly collapse?
2. Were their positions **much worse** than any one of the investment banks which survived? **Not really!**
3. Perhaps this was due to **self-fulfilling beliefs**. Investors became concerned that they would fail, and so rushed to withdrew assets.
4. But if **governments bailout banks** from failure, will not this procedure reduce social welfare because tax-payers are paying for the madness of the crowd or the mistakes of irresponsible bankers?
5. We need tough regulation and supervision.

Two crucial concepts we need

In order to understand the Diamond–Dybvig model, we need to master two crucial concepts

1. Attitudes towards risk: risk aversion, risk loving, risk neutral
2. Strategic behavior (games): Nash Equilibrium

II - Attitudes towards risk

Attitudes towards risk

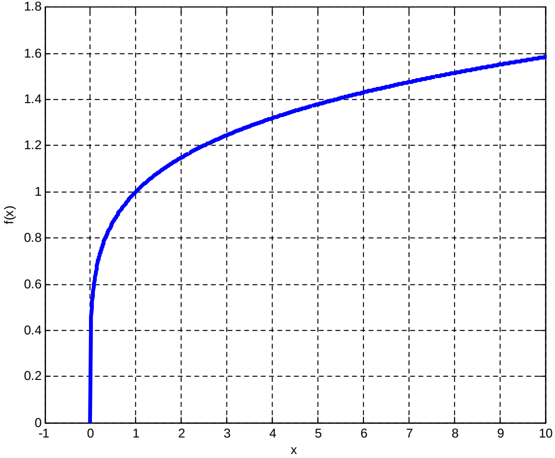
1. Choose between two possible outcomes:
   1. **No game**: I will give you €100 (for sure)
   2. **Play the game**: I will flip a fair coin. If heads I will give you €200; if tails I will give you €0
2. If you prefer the outcome of "No game" over that of "Play the game" you are **risk averse**
3. If you are indifferent between the two outcomes, you are **risk neutral**
4. If you prefer the outcome of "Play the game" over that of "No game", you are a **risk lover**

Risk aversion

An individual is risk averse is he prefers to receive the expected value of a lottery to playing the lottery

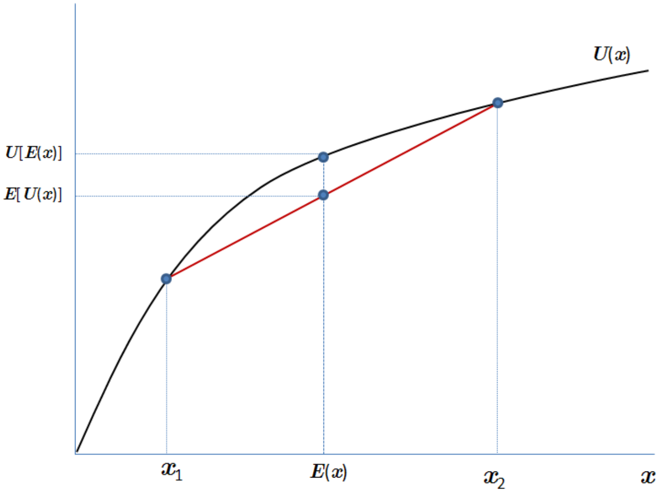
Risk aversion

Payoff function (of the sure outcome) is convex. For example:
$U = f(x) = x^{0.2}$



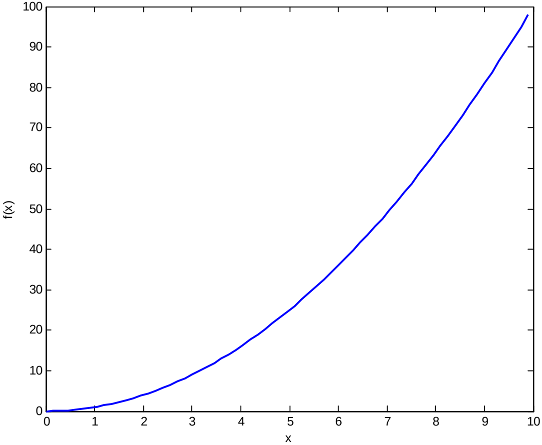
Risk aversion (cont.)

Notice that in this case: $U[E(x)] > E[U(x)]$



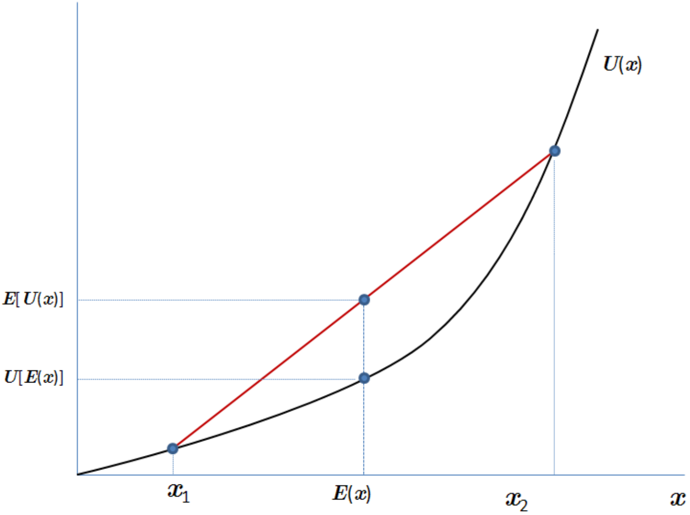
Risk lover

Payoff function (of the sure outcome) is concave. For example:
$U = f(x) = x^2$



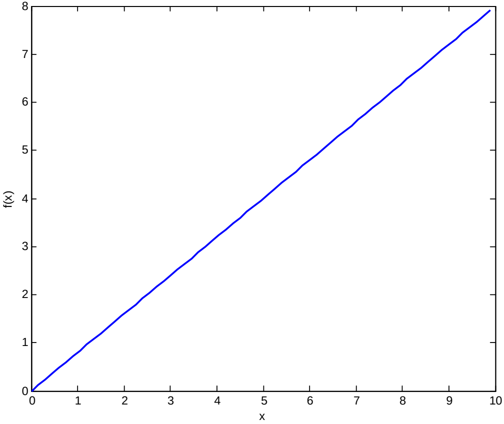
Risk lover (cont.)

Notice that in this case: $U[E(x)] < E[U(x)]$



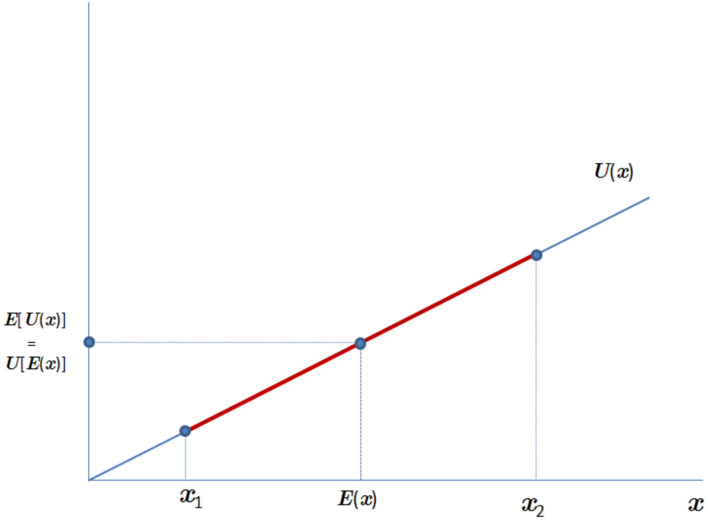
Risk neutral

Payoff function (of the sure outcome) is linear. For example:
$U = f(x) = 0.8x$



Risk neutral (cont.)

Notice that in this case: $U[E(x)] = E[U(x)]$.



III - Introduction to game theory

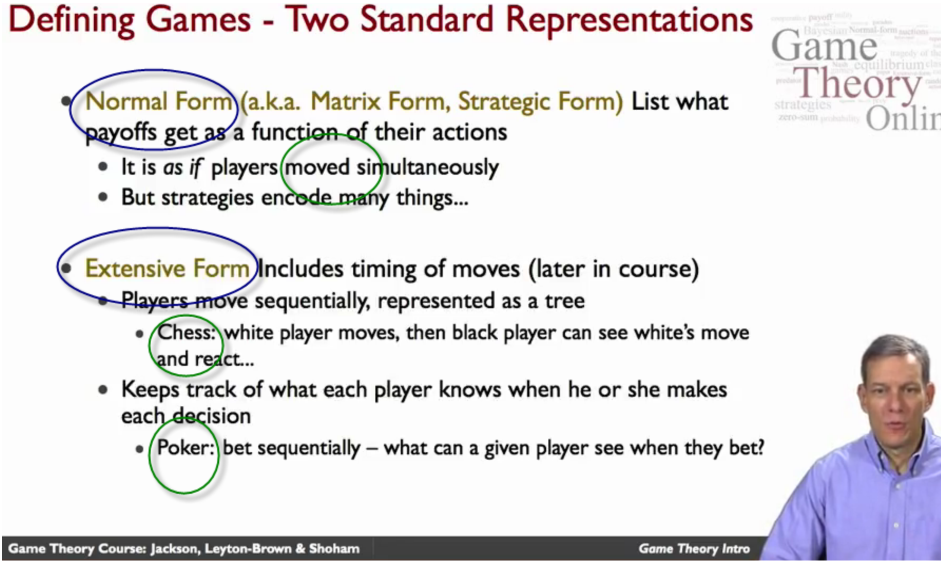
What is game theory?

1.1 Game Theory, Rationality, and Intelligence

Game theory can be defined as the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory provides general mathematical techniques for analyzing situations in which two or more individuals make decisions that will influence one another's welfare. As such, game theory offers insights

Roger B. Myerson (1991). *Game Theory: Analysis of Conflict*, Harvard University Press, page 1.

Two standard representations of games



Normal/Strategic Form Games

- Used to model situations in which players choose strategies without knowing the strategy choices of other players (they move simultaneously)

A strategic form game is composed of

1. A set of players
2. A set of actions/strategies for each player
3. A payoff function for each player

- An **outcome** is a collection of strategies, one for each player.
  - Also known as a strategy profile

An example: price competition

1. Consider two firms selling a similar good
2. Each will independently choose High or Low price
3. If both firms choose High: Each gets 10
4. If both choose Low: Each gets 5
5. If one chooses High, the other Low: High gets 2, Low gets 15

Major ingredients of the Diamond-Dybvig model

- The payoff matrix for this game is

		Player 2	
		High	Low
Player 1	High	$10, 10$	$2, 15$
	Low	$15, 2$	$5, 5$

1. What should Player 1 play?
2. Does that depend on what he thinks Player 2 will do?
3. Low is an example of a dominant strategy
4. It is optimal independent of what other players do
5. How about Player 2?
6. (Low, Low) is a dominant strategy equilibrium

Dominant Strategies

Dominant Strategy: strictly dominant

A strategy A strictly dominatesanother strategy B if it yields a strictly higher payoff irrespective of how the other players play.

Dominant Strategy: weekly dominant

A strategy A weakly dominates$B$ if it never does worse than B and sometimes does strictly better.

Dominant Strategy Equilibrium

If every player has a (strictly or weakly) dominant strategy, then the corresponding outcome is a (strictly or weakly) dominant strategy equilibrium.

Dominant Strategy Equilibrium: example

T \ W	H	L
H	$10, 10$	$2, 15$
L	$15, 2$	$5, 5$

- L strictly dominates H
- (L,L) is a strictly dominant strategy equilibrium

T \ W	H	L
H	$10, 10$	$5, 15$
L	$15, 5$	$5, 5$

- L weakly dominates H
- (L,L) is a weakly dominant strategy equilibrium

Dominant Strategy Equilibrium vs Nash Equilibrium

1. A reasonable solution concept
2. It only demands the players to be rational
3. It does not require them to know that the others are rational too
4. But it does not exist in many interesting games
5. In many games you may have not a dominant strategy at all.
6. What to do?
7. **Nash equilibrium**

Nash Equilibrium

Nash Equilibrium

Nash equilibrium is a strategy profile (a collection of strategies, one for each player) such that each strategy is a best response (maximizes payoff) to all the other strategies

1. Nash equilibrium is self-enforcing: no player has an incentive to deviate unilaterally
2. One way to find Nash equilibrium is to first find the **best response correspondence** for each player
3. Best response correspondence gives the set of payoff maximizing strategies for each strategy profile of the other players
4. ... and then find where they "intersect"

The effort game

- Consider a project the quality of which depends on the smallest effort allocated to it
- Two effort level - two player version
- The payoff matrix is

	Low	High
Low	7, 7	7, 1
High	1, 7	13, 13

- Is there a dominated strategy?
- No.
- But what about, if each **best responds** to what he thinks the other player will play?

The effort game: Nash Equilibrium

	Low	High
Low	$7, 7$	$7, 1$
High	$1, 7$	$13, 13$

1. Remember we want to obtain an equilibrium.
2. If player 1 **expects** player 2 to choose Low, what is her best strategy (best response)?
3. If player 2 expects player 1 to choose Low what is its best response?
4. (Low, Low) is an outcome such that
   1. Each player best responds, given what she believes the other will do
   2. Their beliefs are correct
5. It is a Nash equilibrium
6. Is (Low,Low) **the only** Nash equilibrium?

The effort game: Two Nash Equilibria

	Low	High
Low	$\underline{7}, \underline{7}$	$7, 1$
High	$1, 7$	$\underline{13}, \underline{13}$

1. Payer 1 best response to Low is Low
2. Her best response to High is High
3. Similarly for player 2
4. Best response correspondences intersect at (Low, Low) and (High, High)
5. These two strategy profiles are the two Nash equilibria of this game
6. We would expect in the long-run one of these outcomes to prevail
7. How: Risk dominance vs Payoff dominance

IV - The Diamond-Dybvig bank run model

Major ingredients of the Diamond-Dybvig model

1. **Agents.** Consider two parties or agents:
    1. Depositors (consumers)
    2. Banks
2. **Time periods**: there are three periods: $T = 0, 1, 2$.
3. **Production technology**: the economy has a technology that:
    1. converts 1 unit of good at $T = 0$ into R at date $T = 2$.
    2. If technology interrupted at $T = 1$, only returns 1 and nothing is produced at $T = 2$.

Consumers or depositors

1. **Consumers**: large number N of consumers
2. Two types of consumers: *early* (consume in $T = 1$) and *late* (consume in $T = 2$)
3. Each consumer is endowed with **1** unit of good $T = 0$. ("1 unit" just for simplicity)
4. At $T = 0$ agents don't know their type, only know that there is probability p they will be an early consumer.
5. Expected utility:

$$EU = p \cdot U(c_1) + (1 - p) \cdot U(c_2).$$

6. They have two investment possibilities for this 1 unit:
    1. Direct investment (investing in a low liquidity asset)
    2. Making a deposit in a bank (higher liquidity asset)
7. See next figures

Banks

1. **Banks** have both a short term and a long term investment opportunity for the money.
    1. The short term investment (reserves) is locking the money in the vault. This investment returns the exact amount invested: 1
    2. The long term investment returns an amount R at $T = 2$.
2. They promise to pay depositors:
    1. r_1 to early consumers at $T = 1$
    2. r_2 to late consumers $T = 2$

Society with no banks

$$\text{No Banks}\quad \left\{\begin{array}{ccccc} T_0 & & T_1 & & T_2 \\ 1 & \longrightarrow & 1 & \longrightarrow & 0 \\ 1 & \longrightarrow & 0 & \longrightarrow & R \end{array}\right.$$

Society can be better of with a bank

With Banks

$$\begin{array}{ccc} T_0 & T_1 & T_2 \\ 1 & \longrightarrow \; r_1 & \longrightarrow \; r_2 \end{array}$$

$$\boxed{1 < r_1 < r_2 < R}$$

A numerical example: risk aversion

1. Assume that depositors are risk averse, with the following utility function
$$U(c) = 1 - \frac{1}{c}$$
2. Consider the following parameters
$$R = 2, \quad p = 0.25, \quad r_1 = 1.28, \quad r_2 = 1.81$$
3. What gives the highest return:
    1. Direct investment (lower liquidity asset)
    2. Or making a bank deposit (high liquidity investment)
4. Let's see.

A numerical example: risk aversion (cont.)

1. Expected utility of a less liquid asset

$$\begin{aligned} EU &= p \cdot U(c_1) + (1-p) \cdot U(c_2) \\ &= 0.25(1 - \frac{1}{1}) + 0.75 \left(1 - \frac{1}{R}\right) \\ &= 0.25(1 - \frac{1}{1}) + 0.75 \left(1 - \frac{1}{2}\right) \\ &= 0.375 \end{aligned}$$

2. Expected utility of a bank deposit (more liquid investment)

$$\begin{aligned} EU &= p \cdot U(c_1) + (1-p) \cdot U(c_2) \\ &= 0.25(1 - \frac{1}{r_1}) + 0.75 \left(1 - \frac{1}{r_2}\right) \\ &= 0.25(1 - \frac{1}{1.28}) + 0.75 \left(1 - \frac{1}{1.81}\right) \\ &= 0.391 \end{aligned}$$

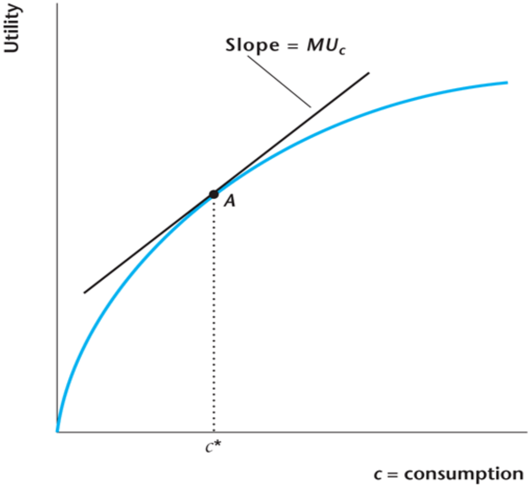
Consumers or depositors

1. Let's concentrate on the explanation provided by Stephen **Williamson** (2011).
2. It is simple and based on graphical analysis
3. Consider that depositors can invest at $T = 0$ in a technology giving a payoff in $T = 2$ equal to
$$R = 1 + r$$
4. Expected utility as usual:
$$EU = p \cdot U(c_1) + (1 - p) \cdot U(c_2)$$

Stephen **Williamson** (2011). *Macroeconomics*, 4th Edition, Pearson, New York. **(chapter 16, pages 591-604).** `Good introduction to the problem of bank runs with some good and simple end-chapter questions to be answered`

Marginal utility and consumption smoothing

Decreasing marginal utility implies a desire to **smooth consumption** between periods $T = 1, 2$



The utility of consumers

1. Expected utility over the two periods is the sum of the utilities in both periods, multiplied (weighted) by the probability that you consume in one of the two periods

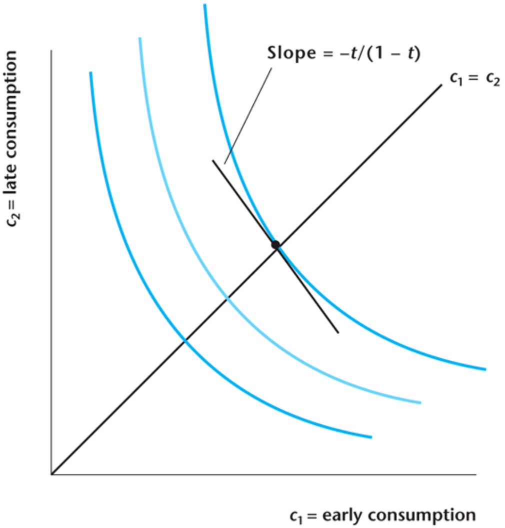
$$EU = p \cdot U(c_1) + (1-p) \cdot U(c_2).$$

2. The marginal rate of substitution of early consumption for late consumption is given by

$$MRS_{c_1,c_2} = -\frac{p \cdot U'_{c_1}}{(1-p) \cdot U'_{c_2}}$$

3. The MRS_{c_1,c_2} tells me how much of c_1 I am ready to give up, in order to have an extra unit of c_2.
4. See figure next slide

The MRS in a figure



The role of banks

1. **If no banks**: all agents invest, and if they are early consumers they get $c_1 = 1$, if late consumers $c_2 = 1 + r$.
2. But the economy can do better: set up a bank to share risk
3. The consumer is better off, because he likes to **smooth consumption**

Deposits contract: the two constraints

1. There are N consumers paying N amounts of resources
2. **First constraint** that a deposit contract must satisfy is:

$$N \cdot p \cdot c_1 = N \cdot x$$

   if only early consumers withdraw at $T = 1$, bank must interrupt a **fraction x of projects**
3. That is: the number of consumers who want to consume in period $t = 1$, Np times their consumption c_1, has to be equal to the fraction of total resources invested which is interrupted, xN
4. **Second constraint** that a deposit contract must satisfy is given by the remaining fraction to pay out to late consumers at 2:

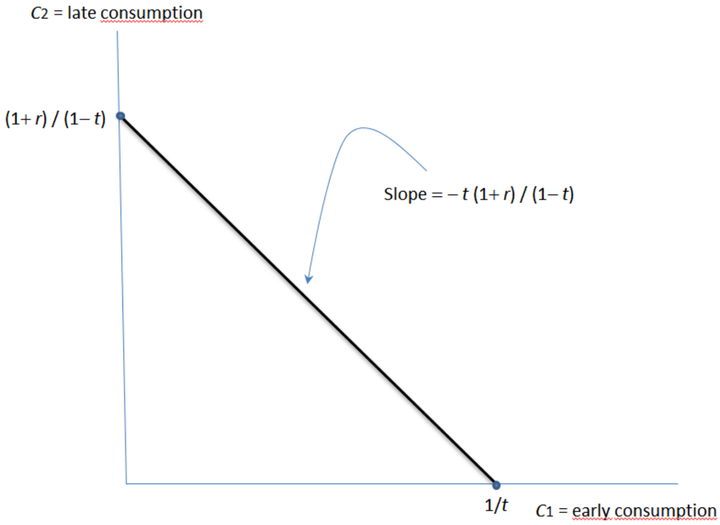
$$N(1-p)c_2 = (1-x)N(1+r).$$

5. That is: The fraction of consumers that does not interrupt, $N(1-p)$ times their consumption c_2 has to be equal to resources available in period 2

Deposits contract: the consolidated constraint

1. We got
$$N \cdot p \cdot c_1 = N \cdot x$$
2. ... and also
$$N(1-p)c_2 = (1-x)N(1+r).$$
3. Combine the two constraints, by cancelling out x and N, to get a **consolidated constraint**:
$$p \cdot c_1 + \frac{(1-p)c_2}{1+r} = 1$$
4. Or in a slightly more useful way
$$c_2 = \underbrace{-\frac{p(1+r)}{1-p}}_{\substack{=\partial c_2/\partial c_1 \\ =\textit{relative price}}} \times c_1 + \frac{1+r}{1-p}$$
5. See next figure

Deposits contract: the consolidated constraint



The equilibrium bank

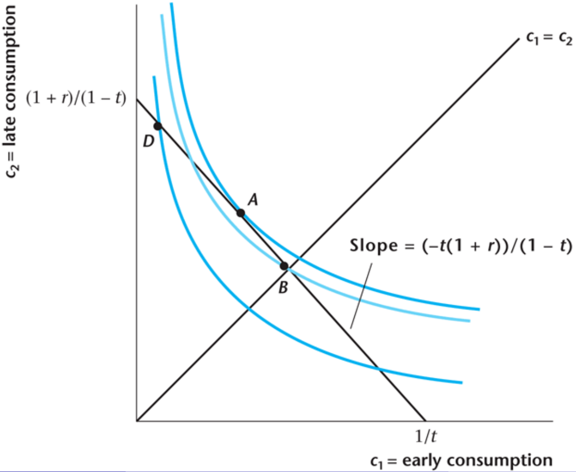
1. There is one (representative) bank making zero (abnormal) profits, assuming free entry
2. This implies that an efficient outcome is the equilibrium
3. The efficient outcome: maximize utility subject to budget constraint of bank

$$MRS_{c_1,c_2} = \textit{Relative Price}$$

   "*Relative Price*" – *The price of future consumption with respect to current consumption*
4. This determines what consumer gets in the two periods
5. See next figure

The Equilibrium Deposit Contract Offered

1. Equilibrium **A** gives the fraction of investments interrupted x.
2. Point **D** is the outcome without a bank: 1 in $T = 1$ and $1 + r$ in $T = 2$



An algebraic derivation of the optimal contract

1. The bank chooses:

$$\max_{c_1,c_2} pU(c_1) + (1-p)U(c_2)$$
$$subject\ to$$
$$c_2 = -\frac{p(1+r)}{1-p}c_1 + \frac{1+r}{1-p}$$

2. The Lagrangean can be written as

$$\mathcal{L} = p \cdot U(c_1) + (1-p)U(c_2) + \lambda\left(-\frac{p(1+r)}{1-p}c_1 + \frac{1+r}{1-p} - c_2\right)$$

   where λ is the the Lagrange multiplier

3. Calculating the First Order Conditions (FOCS) with respect to

$$\begin{aligned} \partial\mathcal{L}/\partial c_1 &= 0 \\ \partial\mathcal{L}/\partial c_2 &= 0 \\ \partial\mathcal{L}/\partial \lambda &= 0 \end{aligned}$$

An algebraic derivation of the optimal contract

1. Calculating the First Order Conditions (FOCS) with respect to $\partial\mathcal{L}/\partial c_1 = 0, \partial\mathcal{L}/\partial c_2 = 0, \partial\mathcal{L}/\partial\lambda = 0$
2. Eliminating the Lagrange multiplier λ from the two first FOCs
3. And we get

$$\underbrace{\frac{p \cdot U'_{c_1}}{(1-p) \cdot U'_{c_2}}}_{=MRS_{c_1,c_2}} = \underbrace{\frac{p(1+r)}{1-p}}_{slope\ of\ consolidated\ constraint}$$

4. Simplifying the result leads to

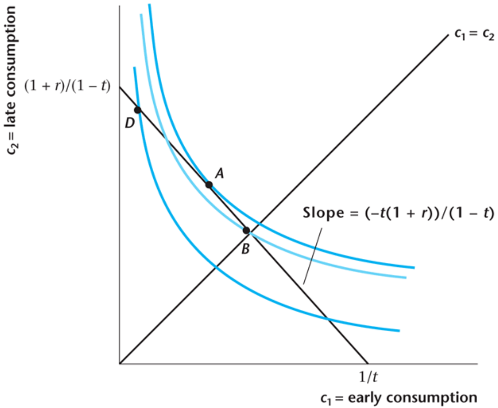
$$U'_{c_1} = (1+r)U'_{c_2}$$

5. But, then, as $(1+r) > 1$, we get

$$U'_{c_1} > U'_{c_2} \Rightarrow c_1 < c_2$$

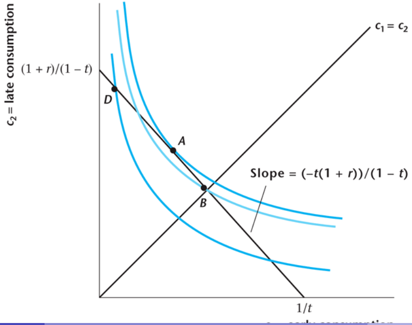
Major point 1

1. In equilibrium **A**, with a bank, there is more consumption in period 1 than in **D** without a bank.
2. This reflects the desire to smooth consumption and, secondly, the possibility to smooth consumption, because there is a bank



Major point 2

1. Point **B** is the outcome with equal consumption in both periods: The MRS is equal to $-p/(1-p)$
2. **Then**: $c_1(A) < c_1(B)$, but $c_2(A) > c_2(B)$
3. This reflects the technological payoff in the economy that keeping the investment until period 2 generates a rate of return of $1+r$.
4. Hence you want to make it optimal for consumers who don't face a shock to wait with consumption till $T=2$, paying them more



Good Equilibrium

1. Consumers **who don't face a shock**, a fraction $1 - p$:
   1. They will wait for consumption until $T = 2$, because they get more in $T = 2$
2. Consumers who face a shock, a fraction (p) :
   1. They do consume in period 1, and forego the rate of return from waiting
3. **This is the good equilibrium: POINT A**

Bad Equilibrium

1. But now **assume that there is a shock** that hits the economy and the consumers's beliefs
2. Suppose that the bank works with a **first come first serve system**: people first in the line for the bank get their deposits first
3. When a $T = 2$ consumer thinks that all other $T = 2$ consumers want to consume in $T = 1$...
4. Then it is **optimal to also queue** in $T = 1$. This gives the probability of at least some payoff, whereas with waiting till $T = 2$, there is for sure nothing left

Bad Equilibrium (cont.)

1. Nothing left ... because the bank only has Nx resources
2. Whereas $(N-1)c_1$ is the amount of consumption $N-1$ agents want to withdraw
3. Remember that $c_1 > 1$, so ...
4. The result is a bank run ... everybody wants their money back ...
5. **This is the bad equilibrium**

Bad and Good Equilibria: an example

1. 4 depositors, each with €1, half need to consume on $T = 1(\ p = 0.5)$
2. The following parameters

$$R = 4, \ \ p = 0.5, \ \ r_1 = 1.5, \ \ r_2 = 2$$

3. Suppose on $T = 1$, 2 depositors withdraw
4. Bank is left with €1
5. This brings €4 on $T = 2$, which the bank returns to the other two depositors
6. What happens if 3 depositors withdraw on $T = 1$?
7. What happens if 4 depositors withdraw on $T = 1$?
8. Let's see.

Bad and Good Equilibria: an example (cont.)

1. If 3 depositors withdraw on $T = 1$,then:
   1. The bank goes bankrupt
   2. They each get $4/3$, the other gets nothing
2. If 4 depositors withdraw on $T = 1$,then:
   1. The bank goes bankrupt
   2. They each get 1
3. Now we can build our payoff matrix
4. See next slide

Bad and Good Equilibria: an example (cont.)

1. The game between two $T = 2$ depositors is given by the matrix

	W	N
W	$1, 1$	$4/3, 0$
N	$0, 4/3$	$2, 2$

1. We have two Nash equilibria:
    1. (W,W): the bad equilibrium
    2. (N,N): the good equilibrium
2. Starting from good equilibrium a rumor that the bank is not doing well could cause a run
3. This is so even if most know it not to be true
4. You only need people believe that the others will withdraw

Possible solution: deposit insurance

1. **The bank run is an equilibrium**, because it is optimal to queue up given that others do the same
2. If the **government guarantees** the value of all deposits, the bad equilibrium disappears
3. It is **not optimal anymore** for an individual $T = 2$ consumer to queue up in $T = 1$, even if all others queue up
4. She will get her money in $T = 2$, as it is guaranteed by the government. And it is more than what she could get in period 1
5. So, she does not queue up and **the bank run does not take place**

Optimality of Deposit Insurance: Too Big To Fail Doctrine

1. The government has a direct interest in protecting deposit holders, because bank runs lead to losses for the deposit holders
2. But a **wider reason** for deposit insurance is that a bank run might be the onset for further problems
3. If a bank collapses, other banks that invested in that bank may also collapse, because the value of their assets becomes too small
4. Hence, many financial institutions are **'too big to fail'**

Problem of Deposit Insurance: Moral Hazard

1. If banks know that deposits are insured, it becomes attractive to **take excessive risk** (moral hazard):
2. The government guarantees the deposits anyway, so deposit holders have no incentive to look for a careful bank and will only **look for the bank with the highest rate of return**
3. As seen in previous class, the **banks with higher ROE** are those that take more risky investments
4. Therefore, deposit insurance has to go along with **regulation and supervision** to prevent excessive risk taking

Bibliography (compulsory reading)

- Stephen **Williamson** (2011). *Macroeconomics*, 4th Edition, Pearson, New York. **(chapter 16, pages 591-604).** `Good introduction to the problem of bank runs with some good and simple end-chapter questions to be answered`

- Douglas Diamond (2007). Banks and Liquidity Creation: A Simple Exposition of the Diamond-Dybvig Model, *Federal Reserve Bank of Richmond Economic Quarterly*, 93(2), 189–200. `Read this paper`

Bibliography (optional reading)

- Franklin **Allen** and Douglas **Gale**, 2007. *Understanding Financial Crisis*, Oxford University Press, Oxford. `(chapter 3). The best overview of the problem at an intermediate level: covers all major theories of bank runs`

- Jean-Charles Rochet (2007). *Why Are there So Many Banking Crises? The Politics and Policy of Bank Regulation*, Princeton University Press, Princeton. `(chapter 2). The best overview of the problem at a relatively more advanced tretment: covers all major theories of bank runs`

- Xavier **Freixas** and Jean-Charles **Rochet** (2008). *Microeconomics of Banking*, 2nd ed., MIT Press, Cambridge, Mass.. `See chapter 7, which has a somewhat similar tone to the previous reference.`